

Inertial effects on Berry's phase of neutrino oscillations

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Abstract. The Berry phase of mixed states, here neutrino oscillations, is calculated in an accelerating and rotating reference frame. It turns out to depend on the vacuum mixing angle, the mass-squared difference and on the coupling between the momentum of the neutrino and the spinorial connection. Berry's phase for solar neutrinos and its geometrical aspects are also discussed.

1 Introduction

Particle interferometry has been recognized as a tool for performing delicate measurements of physical quantities that can be related to the difference in phase of the interfering beams. Moreover, the observation of quantum mechanical phases provides accurate information about quantities determining the phase shift (like, for example, the flux of a magnetic field) and allows one to test the quantum behavior of systems. A remarkable contribution to this topic has been given by Berry in his pioneering paper. In [1], he showed that a quantum state acquires an additional phase (Berry's phase) β^g , which is related to the geometry of parameter space, besides the *normal* dynamical phase ϕ . He considered a non-degenerate quantum system in an initial eigenstate of a Hamiltonian which varies adiabatically through a circuit C in the parametric space. The state evolves under the Schrödinger equation remaining in each instant an eigenstate of the Hamiltonian.

Aharonov and Anandan [2] reformulated and generalized Berry's result by disregarding the parameter space and considering an arbitrary, not necessarily adiabatic, cyclic evolution in the projective Hilbert space $\mathcal{P}(\mathcal{H})$, which is the space of one-dimensional subspaces (also called rays) of an appropriate Hilbert space \mathcal{H} (see also [3]).

Nevertheless, in all formulations, Berry's results are based on non-relativistic quantum mechanics and thus are not covariant. The covariant generalization of Berry phase was derived in [4] where, by using the proper time method [5], the authors showed that

$$\begin{aligned}\beta^g &= i \oint_C \langle \tilde{\Psi}(\lambda) | \frac{\partial}{\partial \lambda_a} | \tilde{\Psi}(\lambda) \rangle d\lambda_a \\ &= \phi + i \oint_C \langle \Psi(\lambda) | \frac{\partial}{\partial \lambda_a} | \Psi(\lambda) \rangle d\lambda_a,\end{aligned}\quad (1)$$

where λ_a are the evolution parameters depending on the proper time, the state $\tilde{\Psi}(\lambda)$ satisfies the cyclic condition, $|\tilde{\Psi}(\Lambda + \lambda)\rangle = |\tilde{\Psi}(\lambda)\rangle$, and $|\Psi(\Lambda + \lambda)\rangle = e^{i\phi}|\Psi(\lambda)\rangle$.

On the experimental side, a series of striking experiments have also been carried out and their results provide direct or indirect evidence for the existence of such a geometric phase [6].

The aim of this paper is to derive the geometrical phase attributed to the cyclic evolution of a mixed state, as, for example, neutrino oscillations. This result is obtained in a non-inertial reference frame, i.e. for an accelerating and rotating observer. As is well known, the effects of the acceleration and the angular velocity are relevant in interferometry experiments. In fact, by using an accelerated neutron interferometer, Bonse and Wroblewski derived the predicted phase shift of quantum systems [7]. Because of the validity of the equivalence principle, one expects that this effect occurs also in a gravitational field, as has been verified in [8]. Besides, Mashhoon has derived a coupling of the neutron spin to the rotation of a non-inertial reference frame [9] from an extension of the hypothesis of locality. In [10], the neutron Sagnac effect has been found using an angular velocity of about 30 times that of Earth. The spin-rotation and the spin-acceleration contributions to the helicity precession of fermions has been calculated in [11], and recently, the inertial effects on neutrino oscillations have been analyzed in [12].

We find that the geometrical phase, when evaluated in a non-inertial reference frame, does depend on the parameters characterizing the physics of neutrino oscillations, i.e. the vacuum mixing angle and the mass-squared difference, and the ones characterizing the background geometry, the acceleration and angular velocity in our case. Besides, dependence on the energy of neutrino also appears. The quantum phase shift induced by the non-trivial topology of the background space-time has, as we will see,

interesting geometrical consequences in connection to the *curvature* of the parameter space of the states.

The contents of this paper are as follows. In Sect. 2, we review the main features of neutrino evolution in an accelerating and rotating frame (see [12]). The Berry phase in such a frame is calculated in Sect. 3. A discussion and our conclusions are in Sect. 4.

2 Neutrino evolution in a non-inertial frame

In [13], the neutrino phase is generalized in the following way:

$$|\psi(\lambda)\rangle = \sum_j U_{fj} e^{i \int_{\lambda_0}^{\lambda} P \cdot p_{\text{null}} d\lambda'} |\nu_j\rangle, \quad (2)$$

where flavor and mass indices are indicated by f and by latin letters, respectively. U_{fj} are the matrix elements transforming the flavor and mass bases, P is the four-momentum operator generating space-time translations of the eigenstates and $p_{\text{null}}^\mu = dx^\mu/d\lambda$ is the tangent vector attached to the neutrino worldline x^μ , parameterized by λ ; $x^\mu = (x^0, \vec{x})$ are the local coordinates for the observer at the origin. We use natural units.

The momentum operator P_μ , used to calculate the phase of neutrino oscillations, is derived from the mass shell condition

$$(P_\mu + A_{G\mu} \gamma^5)(P^\mu + A_G^\mu \gamma^5) = -M_f^2, \quad (3)$$

where the A_G^μ are related to the spinorial connections appearing into the covariant Dirac equation in curved space-time [14], M_f^2 is the vacuum mass matrix in a flavor basis,

$$M_f^2 = U \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} U^\dagger, \quad U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}. \quad (4)$$

θ is the vacuum mixing angle. Ignoring terms of the order $\mathcal{O}(A_G^2)$ and $\mathcal{O}(A_G M_f)$, one sees that for relativistic neutrinos moving along generic trajectories parameterized by λ the column vector of the flavor amplitude,

$$\chi(\lambda) = \begin{pmatrix} \langle \nu_e | \psi(\lambda) \rangle \\ \langle \nu_\mu | \psi(\lambda) \rangle \end{pmatrix}, \quad (5)$$

satisfies the equation

$$i \frac{d\chi}{d\lambda} = \left(\frac{M_f^2}{2} + p \cdot A_G \gamma^5 \right) \chi. \quad (6)$$

In deriving (6), one uses the relation $P^0 = p^0$ and $P^i \approx p^i$ [13].

In a frame with acceleration \vec{a} and angular velocity $\vec{\omega}$, the components of A_G^μ are [12]

$$A_G^0 = 0, \quad \vec{A}_G = \frac{\sqrt{-g}}{2} \frac{1}{1 + \frac{\vec{a} \cdot \vec{x}}{c^2}} \left\{ 2 \frac{\vec{\omega}}{c} - \frac{1}{c^2} [\vec{a} \wedge (\vec{x} \wedge \vec{\omega})] \right\}, \quad (7)$$

so that (6) becomes

$$i \frac{d}{d\lambda} \begin{pmatrix} a_e \\ a_\mu \end{pmatrix} = \mathcal{T} \begin{pmatrix} a_e \\ a_\mu \end{pmatrix}. \quad (8)$$

$(-g)^{1/2}$ is the determinant of the metric tensor of the accelerating and rotating frame [15], $a_f \equiv \langle \nu_f | \psi(\lambda) \rangle$, $f = e, \mu$ and the matrix \mathcal{T} is defined as

$$\mathcal{T} = \begin{bmatrix} -(\Delta/2) \cos 2\theta & (\Delta/2) \sin 2\theta - \vec{p} \cdot \vec{A}_G \\ (\Delta/2) \sin 2\theta - \vec{p} \cdot \vec{A}_G & (\Delta/2) \cos 2\theta \end{bmatrix} \quad (9)$$

up to the $(m_1^2 + m_2^2)/2$ term, proportional to the identity matrix. Here $\Delta \equiv (m_2^2 - m_1^2)/2$. We restrict our considerations to the flavors e, μ , but this analysis works also for different neutrino flavors. In order to determine the mass eigenstates $|\nu_1\rangle$ and $|\nu_2\rangle$, one has to diagonalize the matrix \mathcal{T} . Using the standard procedure, one writes the mass eigenstates as a superposition of flavor eigenstates,

$$|\nu_1\rangle = \cos \tilde{\theta} |\nu_e\rangle - \sin \tilde{\theta} |\nu_\mu\rangle, \quad (10)$$

$$|\nu_2\rangle = \sin \tilde{\theta} |\nu_e\rangle + \cos \tilde{\theta} |\nu_\mu\rangle,$$

where the mixing angle $\tilde{\theta}$ is defined in terms of the vacuum mixing angle θ :

$$\tan 2\tilde{\theta} = \frac{\Delta \sin 2\theta - 2\vec{p} \cdot \vec{A}_G}{\Delta \cos 2\theta}. \quad (11)$$

We note that $\tilde{\theta} \rightarrow \theta$ as $\vec{A}_G \rightarrow 0$ (i.e. $\vec{a} \rightarrow 0, \vec{\omega} \rightarrow 0$). The eigenvalues of the matrix (9) are

$$\tau_{1,2} = \pm \sqrt{\frac{\Delta^2}{4} \cos^2 2\theta + \left[\frac{\Delta}{2} \sin 2\theta - (\vec{p} \cdot \vec{A}_G) \right]^2}. \quad (12)$$

In the basis of the mass eigenstates, we have $|\psi(\lambda)\rangle = a_1(\lambda)|\nu_1\rangle + a_2(\lambda)|\nu_2\rangle$, so that (8) assumes the form

$$i \frac{d}{d\lambda} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} \tau_1 & 0 \\ 0 & \tau_2 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad (13)$$

where $a_i = \langle \nu_i | \psi(\lambda) \rangle$, $i = 1, 2$, and

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \tilde{U} \begin{pmatrix} a_e \\ a_\mu \end{pmatrix}, \quad \tilde{U} = \begin{pmatrix} \cos \tilde{\theta} & \sin \tilde{\theta} \\ -\sin \tilde{\theta} & \cos \tilde{\theta} \end{pmatrix}. \quad (14)$$

We have used the *adiabatic* condition $d\tilde{\theta}/d\lambda \approx 0$ in order that (13) is a diagonal matrix, i.e., we are neglecting the variations of the acceleration and the angular velocity with respect to the affine parameter λ , in comparing to their magnitude. Equation (13) implies

$$a_k(\lambda) = a_k(\lambda_0) e^{\alpha_k(\lambda)}, \quad \alpha_k(\lambda) \equiv -i \int_{\lambda_0}^{\lambda} \tau_k d\lambda' \quad (15)$$

with $k = 1, 2$. For the initial condition $|\psi(\lambda_0)\rangle = |\nu_e\rangle$, the state $|\psi(\lambda)\rangle$ is

$$|\psi(\lambda)\rangle = [\cos \theta_0 \cos \tilde{\theta} e^{-i\tau_1 \lambda} + \sin \theta_0 \sin \tilde{\theta} e^{-i\tau_2 \lambda}] |\nu_e\rangle + [-\cos \theta_0 \sin \tilde{\theta} e^{-i\tau_1 \lambda} + \sin \theta_0 \cos \tilde{\theta} e^{-i\tau_2 \lambda}] |\nu_\mu\rangle, \quad (16)$$

where $\theta_0 = \tilde{\theta}(\lambda_0)$ and the adiabaticity condition has been used in (15). Therefore, accelerating and rotating observers will experience a flavor oscillation of neutrinos. For example, the probability to observe an electron neutrino is readily calculated from (16) and is given by

$$|\langle \nu_e | \psi(\lambda) \rangle|^2 = \cos^2(\theta_0 + \tilde{\theta}) \sin^2 \alpha + \cos^2(\theta_0 - \tilde{\theta}) \cos^2 \alpha. \quad (17)$$

3 Inertial effects on Berry's phase

In this section, the Berry phase is calculated in an accelerating and rotating frame. For convenience, let us invert (10):

$$\begin{aligned} |\nu_e\rangle &= \cos \tilde{\theta} |\nu_1\rangle + \sin \tilde{\theta} |\nu_2\rangle, \\ |\nu_\mu\rangle &= -\sin \tilde{\theta} |\nu_1\rangle + \cos \tilde{\theta} |\nu_2\rangle. \end{aligned} \quad (18)$$

Repeating the previous analysis for the state $|\nu_e(\lambda)\rangle$, we see that (8) can be read, formally, as a Schrödinger equation, the matrix \mathcal{T} playing the role of the effective Hamiltonian. Then the neutrino state $|\nu_e\rangle$ evolves according to the relation

$$\begin{aligned} |\nu_e(\lambda)\rangle &= e^{-i\mathcal{T}\lambda} |\nu_e\rangle, \\ &= e^{-i\tau_1\lambda} \cos \tilde{\theta} |\nu_1\rangle + e^{-i\tau_2\lambda} \sin \tilde{\theta} |\nu_2\rangle, \end{aligned} \quad (19)$$

τ_1 and $\tau_2 = -\tau_1$, defined in (12), being the eigenvalues of \mathcal{T} corresponding to the eigenkets $|\nu_1\rangle$ and $|\nu_2\rangle$, respectively (see (13)). In a cyclic evolution, i.e. from λ to $\Lambda + \lambda$, the final state $|\nu_e(\Lambda + \lambda)\rangle$ and the initial state $|\nu_e(\lambda)\rangle$ (see (19)) coincide, apart from a phase factor, provided that, writing $\Lambda = -2\pi/(\tau_1 - \tau_2)$,

$$|\nu_e(\Lambda + \lambda)\rangle = e^{i\phi} |\nu_e(\lambda)\rangle, \quad \phi = \frac{2\pi\tau_1}{\tau_1 - \tau_2} = \pi, \quad (20)$$

Inserting (20) into (1) (for $a = 1$, i.e. the evolution parameter does coincide with the affine parameter), the Berry phase of an electron neutrino moving along a cyclic path can be written as

$$\beta_e^g = \phi + i \int_0^\Lambda \langle \nu_e(\lambda) | \frac{\partial}{\partial \lambda} | \nu_e(\lambda) \rangle, \quad (21)$$

where ϕ is the dynamical phase evaluated in (20). An explicit calculation of (21) yields the result

$$\begin{aligned} \beta_e^g &= 2\pi \sin^2 \tilde{\theta} \\ &= \pi \left[1 - \frac{\Delta \cos 2\theta}{\sqrt{\Delta^2 \cos^2 2\theta + (\Delta \sin 2\theta - 2\vec{p} \cdot \vec{A}_G)^2}} \right], \end{aligned} \quad (22)$$

where (11) and (18) have been used. In a similar way we get the Berry phase of the muon neutrino:

$$\begin{aligned} \beta_\mu^g &= 2\pi \cos^2 \tilde{\theta} \\ &= \pi \left[1 + \frac{\Delta \cos 2\theta}{\sqrt{\Delta^2 \cos^2 2\theta + (\Delta \sin 2\theta - 2\vec{p} \cdot \vec{A}_G)^2}} \right]. \end{aligned} \quad (23)$$

Table 1. Estimation of $|m_2^2 - m_1^2|$ given by (26) as a function of E_ν , $\sin 2\theta$, and with a fixed value of $\omega \sim 7 \cdot 10^{-5}$ rad/s

E_ν (MeV)	$\sin 2\theta$	$ m_2^2 - m_1^2 $ (eV ²)
1	1	10^{-13}
1	10^{-1}	10^{-12}
10	1	10^{-12}
10	10^{-1}	10^{-11}
50 ÷ 60	1	10^{-10}

As becomes clear from (22) and (23), $\beta_{e,\mu}^g$ depend on the vacuum mixing angle, on the mass-squared difference, on the energy of the neutrinos, and on the parameters characterizing, through the spinorial connection, the background geometry: the acceleration and angular velocity. Besides, the quantum shift phase also induces dependence on the energy of the neutrinos. Note that $\beta_e^g + \beta_\mu^g = 2\pi$.

Direct or indirect evidence of the Berry phases (22) and (23) is very difficult to achieve, and to consider how a quantum interferometry experiment could reveal it is beyond the aim this paper. We just analyze some interesting consequences of the above results.

(1) If the condition

$$\Delta \sin 2\theta - 2\vec{p} \cdot \vec{A}_G = 0 \quad (24)$$

holds, then (11), (22) and (23) imply $\tilde{\theta} = n\pi/2$, $n = 0, 1, 2, \dots$, so that

$$\beta_e^g = 0, \quad \text{and} \quad \beta_\mu^g = 2\pi. \quad (25)$$

Furthermore, in the rotating frame with angular velocity ω and acceleration zero, (24) reduces to the form

$$|m_2^2 - m_1^2| \approx \frac{4E_\nu\omega}{\sin 2\theta}, \quad (26)$$

where E_ν is the energy of the ultra-relativistic neutrinos; $E_\nu \approx |\vec{p}|$. Let us analyze (26) for solar neutrinos in the case in which the observer is comoving with the Earth, i.e. its angular velocity is $\omega \sim 7 \cdot 10^{-5}$ rad/s. Results of the mass-squared difference, calculated by using (26) for typical values of the solar neutrino energies and vacuum mixing angle, are reported in Table 1. The agreement with the experimental data comes from neutrinos with energy varying in the range $10 \div 60$ MeV. In this range, in fact, we find a mass-squared difference of the order $10^{-12} \div 10^{-10}$ eV² for a vacuum mixing angle of $10^{-1} \leq \sin 2\theta \leq 1$ (see also [12]).

At present, there is a strong evidence in favor of oscillations of neutrinos and of non-zero neutrino masses. Such results have been found in different experiments: solar neutrino experiments [16], atmospheric neutrino experiments [17], and the accelerator LSND experiment [18]. Recent reports indicate that the best fits in favor of solar neutrino oscillations are obtained for the following case [19]

$$|m_2^2 - m_1^2| \simeq (0.5 \div 1.1) \cdot 10^{-10} \text{ eV}^2, \quad \sin^2 2\theta \simeq 0.67 \div 1,$$

in very good agreement with our results summarized in Table 1.

(2) If the momentum of the neutrino \vec{p} is perpendicular to the spinorial field \vec{A}_G , then the coupling term vanishes and (22) and (23) reduce to

$$\beta_e^g = 2\pi \sin^2 \theta, \quad \beta_\mu^g = 2\pi \cos^2 \theta, \quad (27)$$

i.e. the mixing angle $\tilde{\theta}$ coincides with the vacuum mixing angle θ . Of course, when the angular velocity and the acceleration of the frame are zero, we trivially recover (27) and the results of [20], obtained for a Minkowski background. Equations (22) and (23) represent a generalization of the results obtained in [20].

(3) For ultra-relativistic neutrinos and high values of the acceleration and angular velocity, so that the coupling term is much more greater than the mass-squared difference, the Berry phase (22) and (23) assume the following values: $\beta_e^g \approx \pi$ and $\beta_\mu^g \approx \pi$.

Extension of these results can be done, invoking the equivalence principle, to stationary gravitational fields, these being responsible for the quantum phase shift. The previous analysis applies also to quantum systems involving boson mixing, like the oscillations of $K^0 - \bar{K}^0$.

4 Discussion and conclusions

In this paper, the covariant generalization of the Berry phase proposed in [4] has been applied to problems involving mixed states, like neutrino oscillations. We have calculated the Berry phase of the electron and muon neutrinos in a cyclic evolution with respect to an accelerating and rotating reference frame and we have found that these phases depend on the vacuum mixing angle, the mass-squared difference and on the coupling between the momentum of the neutrinos and the spinorial connection.

The condition of no-mixing expressed by (24), which diagonalizes the *mixing* matrix (9), implies that the Berry phase of the electron and muon neutrinos assume values of 0 or 2π , respectively. Such a condition has been analyzed for observers comoving with the Earth, so that $\omega \sim 10^{-5}$ rad/s and $|\vec{a}| = 0$. In this particular case, the numerical values of the mass-squared difference, calculated using (26) (see Table 1), are compared with the ones of the recent experimental data, showing a very good agreement. It is worth to note that the inertial effects on the Berry phase, hence on the neutrino oscillations, induce a dependence on the orientation of the rotating observer (the direction of the angular velocity) with respect to the direction of the neutrino momenta. It hence implies dependence on the zenith angle, as recently discussed in the framework of the atmospheric neutrino flux [19].

A direct or indirect measurement of the inertial (or gravitational) effects on the Berry phases related to the neutrino particles is at the moment very difficult, considering also the fact that the values of the mass-squared difference of the neutrinos and the vacuum mixing angle are till now open issues. Nevertheless, the Berry phases (22)

and (23) have interesting implications from the geometrical point of view. As already said in the Introduction, the Berry phase is attributed to the holonomy in the parameter space. It appears when a quantum state is parallel transported in the parameter space around a closed path. The final quantum state is rotated, with respect to the initial one, by an angle which can be related to the connection (the Berry connection) $\mathcal{A} = \langle \tilde{\Psi} | d | \tilde{\Psi} \rangle = \mathcal{A}_a d\lambda^a$ appearing in (1), $d = (\partial/\partial\lambda_a)d\lambda^a$ being the exterior derivative in the parameter space. Moreover, one can also define a curvature (the Berry curvature) as the field strength of \mathcal{A} , $\mathcal{F} = d\mathcal{A} = (d\langle \tilde{\Psi} |) \wedge (d | \tilde{\Psi} \rangle)$.

The existence of this phase, associated with the cyclic evolution, is universal in the sense that it is the same for the infinite number of possible motions along the curves in the Hilbert space, which project to a given closed curve in the projective Hilbert space of the rays, and for the possible (effective) Hamiltonians, which rule the evolution of the state along these curves.

In a Minkowski space-time, the Berry phase is related only to the vacuum mixing angle: The quantum vector state is rotated over the angle θ when parallel transported along a closed curve, so that, as discussed above, it can be associated to the curvature in the space of states (see (27) and [20]).

In a non-inertial reference frame, new geometrical features arise, as one can see from (22) and (23):

(a) The parallel transport along a closed curve contains also the contribution due to the spinorial connection A_G^μ , which induces a quantum shift phase, hence an additional *rotation* in the parameter space of the quantum vector state. The total rotation is given by the (mixing) angle $\tilde{\theta}$, so that the Berry curvature turns out to be related both to the vacuum mixing angle and to the spinorial connection, i.e. to the non-trivial geometry of the background space-time.

(b) Due to the coupling term, $\vec{p} \cdot \vec{A}_G$, the geometrical phases (22) and (23) show a dependence on the energy of the neutrinos, losing in such a way the property of universality: different values of the neutrino energies correspond to different Berry's phases.

Such a new geometrical setting follows from the generalization of the quantum mechanical phase (2) in which, as suggested by Stodolsky [24], the metric field appears in the definition of the scalar product. The dependence of the geometrical phase on the energy of the particles and on the parameters characterizing the background geometry is a common aspect of the behavior of a quantum particle in a certain class of four-dimensional stationary space-times, like those associated with a tubular matter source, a slowly moving mass current and a spinning cosmic string [21–23]. In the latter case, for example, it is shown that the *gravitational* geometrical phase of a quantum particle with energy E moving around a spinning cosmic string (with angular velocity J) is $\sim JE$ [23].

As a final comment, we stress that the previous results hold also for a stationary gravitational field, as an obvious consequence of the validity of the equivalence principle. Then, the understanding of the issues concerning the in-

fluence of the gravity on the Berry phase are of interest in view of theoretical arguments which try to construct general relativity as a gauge theory and, in particular, try to formulate the symmetric second-rank tensor field in the framework of a unified gauge approach. In such a way, the gravitational field should play the role of the *electromagnetic* field, in strict analogy with the Bohm–Aharonov effect, and its theoretical (and phenomenological) consequence could help for a better understanding of the quantum nature of particles evolving in non-trivial geometries.

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